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RADIATION IMPEDANCE OF CONES AT HIGH FREQUENCIES

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ABSTRACT

The cavity of concave diaphragms causes a typical variation of the radiation impedance, which has an effect on the performance of the speaker system in the frequency range where the wavelength is comparable with the dimension of the diaphragm. This paper investigates the radiation impedance and its effects on the sound pressure response of the whole driver with the help of a circular symmetric boundary element method for the infinite baffle. The electrical and mechanical properties of the electro dynamical driver are modeled with lumped elements. An approximation for the wide band radiation impedance of cone type diaphragms is given and compared to the exact results from the BEM.

RADIATION INTO FREE SPACE

If the diaphragm vibrates not in vacuum then the motion of the diaphragm causes radiation. The effect of radiation is usually measured as variations of the sound pressure. The pressure magnitude concentrates at the source and tends to decay with distance. At a very large distance the source can be regarded to be a point, which can form the origin of a spherical co-ordinate system. In this so-called far-field case the magnitude of the pressure field decays by 1/distance in radial direction. In polar direction the variations of the pressure field are called directivity, and these variations are typically constant with distance.

The sound pressure field is affected by boundary conditions. In many cases of radiation problems only natural conditions are specified such as the motion of the diaphragm and reflecting walls. Here the gradient of the pressure field must fit to the normal surface velocity. The transition from the source to the far-field is a diffraction-effect. There is no diffraction if the source is a point and if there are no reflectors present. In any other case either reflections or motion of the source will cause the pressure field to diffract. This can yield to a complex field structure, which is usually difficult to calculate. In fact there are only a few arrangements that have know solutions [1]. In any other case numerical approaches such as finite element techniques are the only choice.

It is of advantage to select boundary conditions, which allow standardizing measurements and which are easily to simulate. Loudspeaker drivers are usually measured and simulated when placed in an infinite baffle. The infinite baffle avoids interference with the sound field, which is produced from the alternate diaphragm side. It can be shown that for concave shapes such as the cone, which are embedded in the flat baffle, only diffraction effects caused by the velocity distribution of the source are monitored. In this case the reflection contribution from the baffle is canceled out.

BOUNDARY ELEMENT METHOD

The Boundary Element Method (BEM) is a solving schema, which can be used for the calculation of the sound pressure field caused by arbitrary vibrating boundaries. The BEM makes use of the Helmholtz integral (HI). A special case of the HI is Rayleigh's formula, which is appropriate if structure and source, which represent the acoustical boundaries, are both flat and the baffle can be assumed to be infinite.

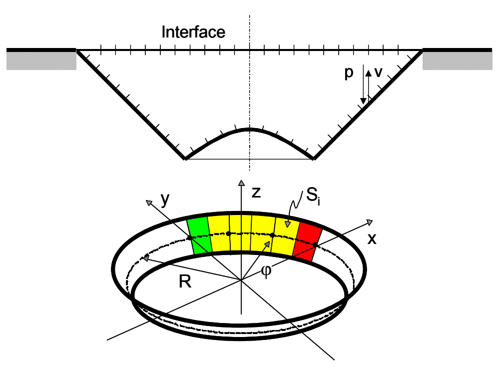


Figure 1: Discretisation of the cone into concentric rings.

If the boundaries are not flat then the complete Helmholtz integral needs to be used. The complete HI adds the reflection term to the Rayleigh formula. Unfortunately this reflection term has unknown parameters as part of its integrand, which yields to the problem of solving an integral equation. The BEM solves for these parameters by discretising the surface of the boundary. Once known, the sound pressure field can be easily calculated anywhere in the radiation space as well as on the boundaries.

A special version of the BEM has been worked out, which takes care of the infinite baffle and makes use of the circular symmetry of cone type diaphragms [2]. The cone is discretised in concentric rings as displayed in figure 1. The numerical integration of the Green functions is performed over each segment. Because of circular symmetry there is only one test-point on each segment, which reduces the order of the system matrix to be solved [3]. It is possible to use a Green-function, which satisfies the boundary conditions imposed by the infinite baffle [1]. In this case there are no segments on the infinite baffle. However, the drawback of the special Green function is that no structures and sources are allowed behind the baffle. A concave diaphragm, which is embedded in the baffle, therefore must be modeled with the help of another domain. The domain of free radiation and of the cavity are then coupled at an interface and solved simultaneously.

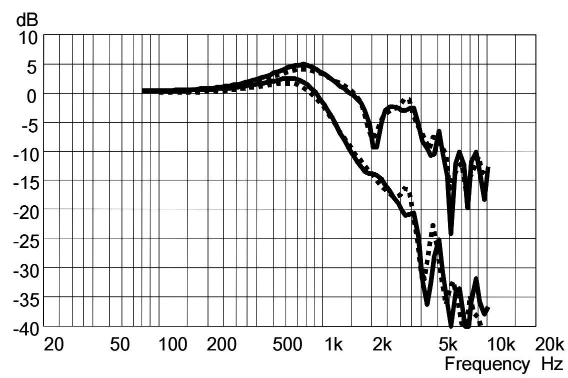


Figure 2: Sound pressure on axis under 90 deg of cone type diaphragm. Comparison of used BEM module vs. commercial FEM/BEM package.

The results of the BEM module have been validated, by comparing the results with those of a commercially available solver. An example is given in figure 2. The BEM module used in this paper models the cavity of the cone as well as the free radiation space with boundary elements, which are solved simultaneously. The reference package on the other hand models the cavity with the help of an acoustic finite element approach, which is then coupled to the free space by using the BEM.

MECHANICAL MODEL

Once implemented, a BEM solver gives the opportunity of modeling radiation in great detail. Each segment is allowed to have arbitrary orientation as well as velocity, either zero for a rigid reflector or any complex value due to damping or mechanical vibration. The next natural step would be to investigate the mechanical dynamics of the diaphragm itself, which is know to perform bending vibration at higher frequencies. Bending waves are known to give raise to high radiation efficiency especially above the coincidence frequency. Also the sound pressure, which is created by the velocity, and which reacts back onto the mechanical system, could be included in the model. Such modeling has been done before and yields interesting results [4]. In these models the circular symmetric diaphragm is usually discretised in similar way as shown in figure 1. The dynamics at the finite elements are then solved by either an energy or transfer-matrix approach.

However, in this paper the diaphragm is assumed to be rigid and the study of the dynamical behavior are left for on-going research.

By using the rigid-body-approximation, the simulation bandwidth is restricted to low and mid frequencies because at high frequencies the radiation from bending waves dominates and superimposes the radiation of the rigid body mode. The advantage of the rigid-body approximation is that no material data are necessary, which often are difficult to obtain. There are only easy to measure geometrical parameters and lumped element parameters describing the mechanical properties of the driver. This paper therefore investigates the response of the rigid body model coupled to a detailed acoustical model by using the BEM.

Rigid body model

The rigid body model for an electro dynamical loudspeaker driver, which radiates directly from an infinite baffle is widely used and well known. Basically the rigid body model is valid only at low frequencies at which all higher modes in the mechanical structure can be ignored. In this case the main parameters can be described with the help of lumped elements.

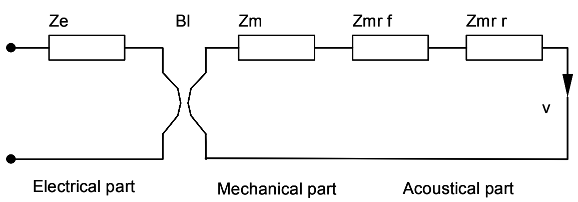


Figure 3: Equivalent circuit of the electro dynamical loudspeaker driver.

In the electrical domain Ze represents the voice coil impedance. Ze is usually modelled with the help of a series combination of a resistance Re and an inductance Le. Either additional components or exponentials are used in order to correct for effects at high frequencies, such as eddy currents in the pole piece etc.

The electro-dynamic motor is symbolised by the gyrator element with force factor Bl. The gyrator transforms the current I through the voice coil wire into the force F, which accelerates the mass Mms of the vibrating assembly (F = Bl·I). The gyrator provides also the feedback voltage U, which the velocity v induces in the voice coil (U = Bl·v). In the above equivalent circuit the velocity is a flow. Force is regarded as a potential. In the rigid-body model the voice coil and the diaphragm is assumed to be infinite stiff and performs motion of one degree of freedom only (axial motion).

The motion of the diaphragm causes radiation, which in turn induces a pressure on the diaphragm. This pressure integrates to a force at the voice coil. Divided by the velocity of voice coil and diaphragm it forms the radiation impedance Z_{mr}:

$$Z_{mr} = \frac{F}{v} \tag{1}$$

In figure 3 Z_{mrf} symbolises the radiation impedance caused by the front radiation and Z_{mrr} represents the impedance of radiation into the space to the rear of the driver. If the diaphragm radiates from an infinite baffle then Z_{mrf} and Z_{mrr} are decoupled. The same is true if the driver is built into an enclosure, then Z_{mrr} represents the enclosure-load. Z_{mr} is caused by the velocity of the moving assembly of the driver and is therefore connected in series to Z_m. Because the motion is restricted to the axial direction, the force F is obtained from a simple projection of the integrated pressure in the axial direction. Taking the sum over all finite elements approximates the integration of the pressure:

$$Z_{mr} = \frac{\sum_i p_i \cdot S_i \cdot \cos(\alpha_i)}{v} \tag{2}$$

p_i is the pressure on the i^th^ cone segment. α_i is the angle of the segment-normal to the axis of rotation, S_i is the area of the segment. v is the velocity in axial direction.

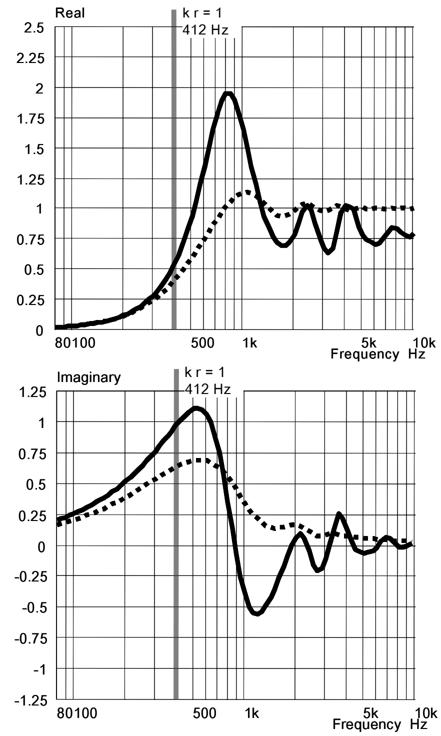


Figure 4: Radiation impedance normalised to the asymptotic value of the real part. Solid: cone-model, dashed: disk-model

The radiation impedance is frequency dependent as the example in figure 4 demonstrates. The top graph of figure 4 displays the radiation resistance and the next graph the radiation reactance. The solid curves are obtained from a cone shaped diaphragm. For comparison included are also the curves of a rigid disk, which can be used for a low frequency approximation of the cone. In this case the diameter of the disk is set equal to the outer diameter of the cone plus half of the suspension assuming a linear velocity profile. The radiation impedance of a rigid disk set in an infinite baffle is derived from Rayleigh's formula and can easily be calculated. The curves of the cone radiation impedance have been calculated with the help of the BEM. However, some approximation formulae are given in the appendix.

The radiation impedance is a complex value, where the real part causes damping to the mechanical system. It is also related to the acoustical power, which is radiated to the far-field and hence, the real part is an indicator for the radiation efficiency. The peak in the transition frequency range of the radiation resistance is typical for a cone.

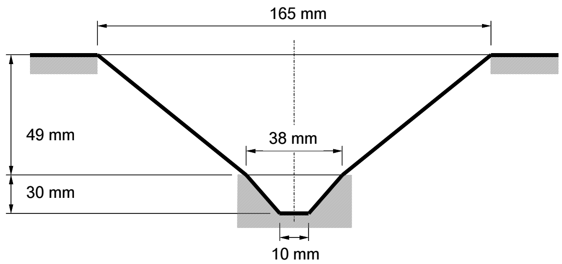
The imaginary part describes the flux of acoustic energy from one part of the diaphragm to another part. Because this energy is only present close to the source it cannot contribute to radiation. The imaginary part of a flat or convex rigid structure in a flat baffle is always mass like. With the cone, however, there is always a frequency range where the radiation reactance has spring type character as well, as indicated by the negative range of the imaginary curve of figure 4. The reactance forms an acoustic load, which couples into the mechanical system of the driver and modifies parameters of the electro-mechanical system such as the resonance frequency. At low frequencies the radiation resistance raises by 12 dB per octave whereas the reactance rises by 6 dB.

There is a particular frequency at which the wavelength is equal to the circumference of the disk as the vertical bar at 412 Hz indicates. At this frequency the wavenumber-radius product is one. Above this frequency the characteristic of the radiation impedance changes. After some rippling the resistance converges to a constant value, whereas the reactance vanishes asymptotically.

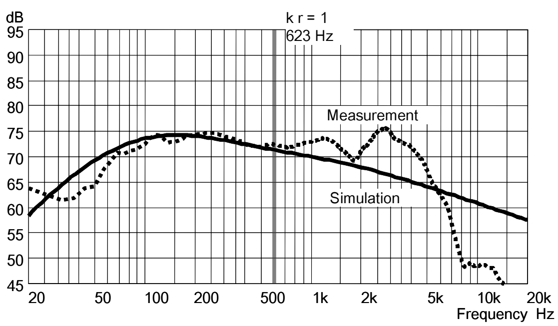
SOUND PRESSURE MEASUREMENTS

It is interesting to compare sound pressure far-field measurements with simulations based on the disk approximation and with simulations, which make use of the boundary element model, as described above. The sound pressure measurement was performed in a small an-echoic chamber. Therefore some reflection effects occur below 100 Hz.

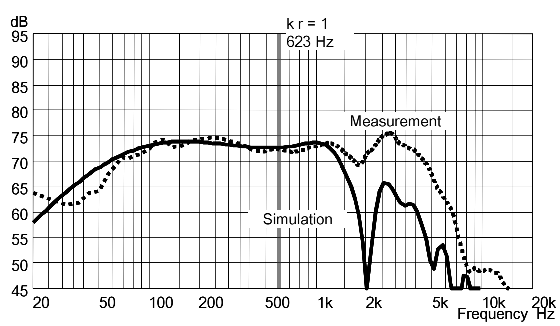
Driver 1



Mms: 26 g, Bl: 9 Tm, Re: 5.3 ohm, Le: 1.9 mH
(Dust cap removed)



Disk approximation



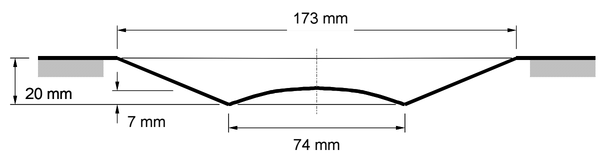
BEM

Figure 5, driver 1: Far-field sound pressure level on-axis.

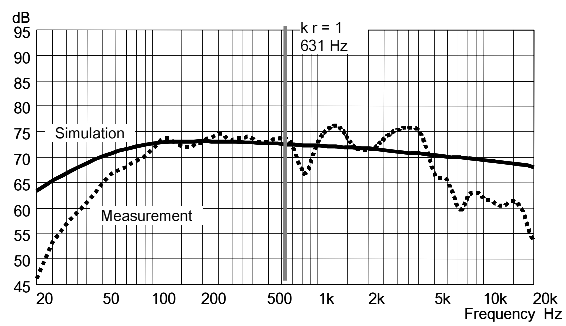
Driver 1 has a steep soft plastic cone. The driver under test had the dust cap removed, which opened a small rigid funnel in the pole piece. This part was included in the BE model. Note, the large voice coil inductance of driver 1.

The top sound pressure response compares the model using the disk approximation with the measurement. The agreement is quiet good up to a frequency of 500 Hz. The range can be extended by more than one octave with the help of the BEM as displayed on the bottom graph. Above 1500 Hz the radiation of shell modes superimposes the radiation of the rigid body mode.

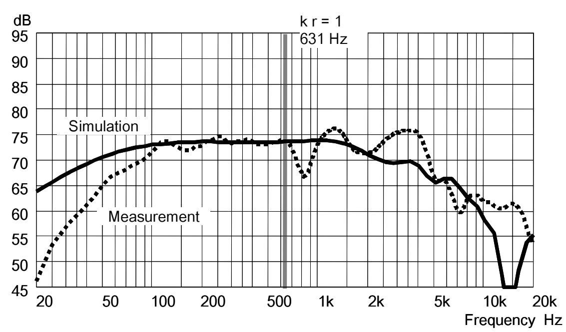
Driver 2



Mms: 22 g, Bl: 7.65 Tm, Re: 6.3 ohm, Le: 0.35 mH



Disk approximation



BEM

Figure 6, driver 2: Far-field sound pressure level on-axis.

Driver 2 is similar to driver 1 but has a very shallow conical diaphragm and a small voice coil inductance Le. Apart from some dips and peaks both simulation models agree very well with the measurement curves, although the BEM curve performs better at higher frequencies.

In general

More experiments have been made, which more or less resembled the above two examples. In general it can be observed that the BEM can give approximately one octave more of simulation bandwidth compared to the disk model. Up to this point only some geometric data have to be added to the list of standard parameters of the driver. The simulation model gives confidence up to a wavenumber-radius product of $k \cdot r = 2 \ldots 2.5$.

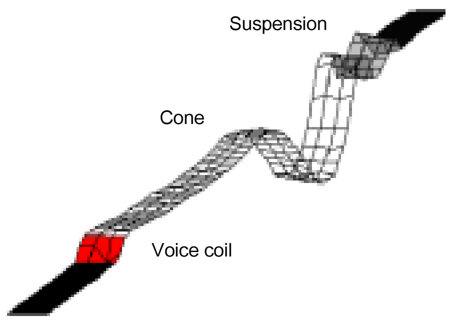


Figure 7, driver 1: Laser vibrometer scan of cone section from centre to rim at 2000 Hz

At higher frequencies radiation from bending vibration of the diaphragm shell can amplify the response on-axis dramatically. For illustration driver 1 was scanned with the help of a laser vibrometer as displayed in figure 7. The black part on the bottom left side is part of the pole piece as sketched in figure 5. Then comes the voice coil followed by the cone. Top right belongs to the suspension part. The frequency is 2 kHz and falls in the region of high bending mode density with strong motion at a band near the outer rim of the cone, which produces strong radiation and causes the amplification above 1500 Hz.

These variations are not covered by the BE approach. Sometimes the disk approximation seems to give a fairly good result at higher frequency. But in this case the model does not reflect the underlying physics. The reason is that the cone cavity couples in its own modal behavior, which acts on the radiation like a low-pass at high frequencies. Over a certain frequency range this effect is somewhat compensated by the radiation of activated mechanical modes in the diaphragm. The exact trace of the response curve can be controlled with the help of the geometry of the cone as well as by selecting the material of the diaphragm, surround etc.

APPROXIMATION OF RADIATION IMPEDANCE

Although the application of the full BEM to the radiation problem of the cone yields a confident simulation for the low and mid-frequencies, it is very involved in the mathematical formulation and software implementation. An intermediate step between the simple disk model and the boundary element schema can be achieved by a simple modification of the disk model, which models the radiation impedance of a rigid cone shaped diaphragm in an infinite baffle. This approximation is based on the driving point impedance of an acoustic duct in fundamental mode.

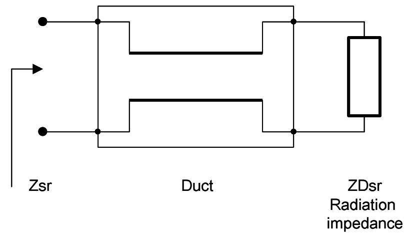


Figure 8: Equivalent circuit of open radiating duct

The open duct terminates in an infinite baffle and is loaded by the radiation impedance of a disk in an infinite baffle [5]. The driving point impedance of this combination as displayed in figure 8 is:

$$Z_{sr} = \frac{Zd_{sr} \cdot \cos(k \cdot L) + i \cdot \sin(k \cdot L)}{\cos(k \cdot L) + i \cdot Zd_{sr} \cdot \sin(k \cdot L)} \quad (3)$$

Z_{sr} is the specific radiation impedance and relates to the mechanical impedance by:

$$Z_{mr} = \rho \cdot c \cdot S \cdot Z_{sr} \quad (4)$$

with ρ density of air, c sound velocity, S duct-cross section area, $k = \omega/c$ is the free-space wavenumber and L is the length of the duct. Zd_{sr} is the specific radiation impedance of a disk in an infinite baffle.

Z_{mr} can be used to approximate Z_{mrf} in the equivalent circuit of figure 3. The examples given in the appendix compare the results of equation (3) with those obtained from the BEM. The curves can be made close by just adjusting the length parameter, L in equation (3).

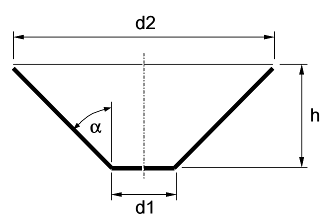


Figure 9: Used dimension parameter of cone

The effective length, L seems to depend on the ratio of outer diameter d2 to the inner diameter d1. In the first series the ratio is $d2/d1 = 2$ and $L = 0.3 \cdot h$, where h is the height of the cone. In the second series the ratio $d2/d1 = 8$ and $L = 0.5 \cdot h$. The end-correction values used in the two series were found in a heuristic way. The exact relation is unclear. However, linear interpolation might be applied for intermediate ratios.

CONCLUSIONS

The application of the boundary element method (BEM) to the radiation problem of conical shaped diaphragms results in a confident model for the low and mid frequency range where the radiation of the rigid body mode of the mechanical system of the driver dominates. In this case only some geometrical data have to be added to the list of lumped element parameter, i.e. no material parameter are necessary. Examples of the sound pressure response of the simulation vs. measurement are given. The coupling of the mechanical and acoustical system is expressed with the help of the radiation impedance. Example curves are given for a variety of cone shapes. These examples are compared to an easy calculable approximation for the cone radiation impedance.

Special thanks are due to Dr. Neil Harris, Dr. Graham Bank and Christien Ellis for their help in the analysis and measurements.

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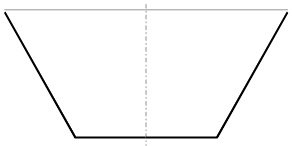
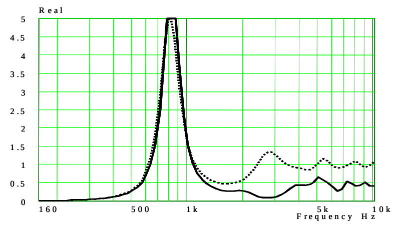
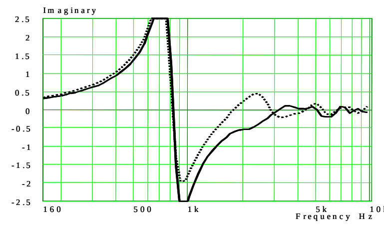
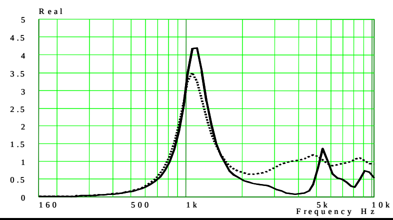
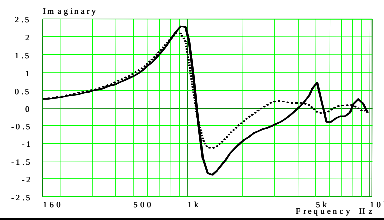
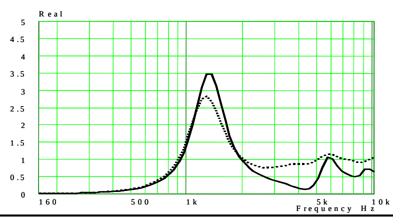
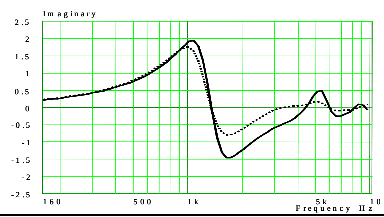
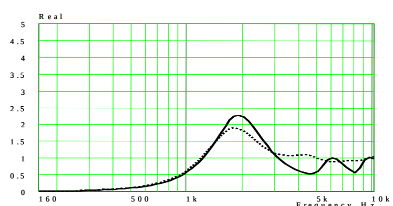
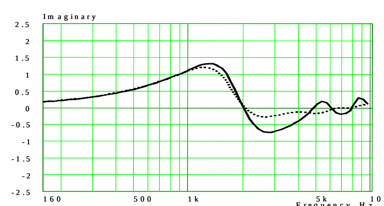
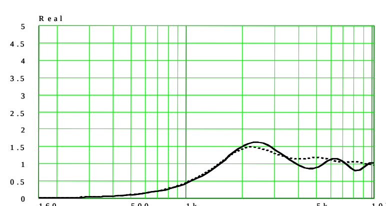
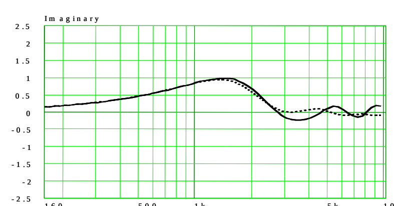
APPENDIX

Approximation of radiation impedance: First series of examples

d2 = 100 mm, d1 = 50 mm, d2/d1 = 2, L = 0.3·h

Parameter	Real part	Imaginary part
 h = 100 mm α = 26.56 º		
 h = 65.5 mm α = 37.36 º		
 h = 50 mm α = 45 º		
 h = 25 mm α = 63.43 º		
 h = 12.5 mm α = 76.0 º		

Approximation of radiation impedance: Second series of examples

d2 = 100 mm, d1 = 12.5 mm, d2/d1 = 8, L = 0.5·h

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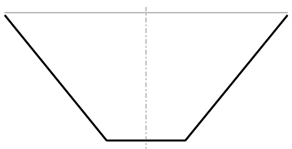
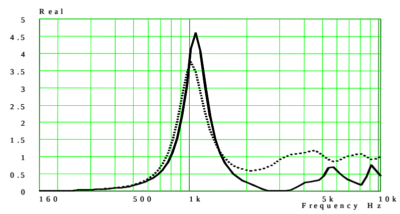


h = 100 mm

α = 41.2 º</td><td>

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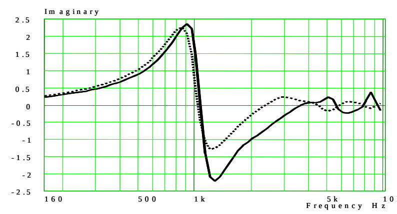
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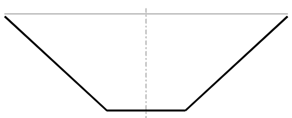

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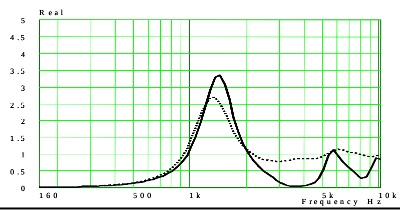
α = 53.2 º</td><td>

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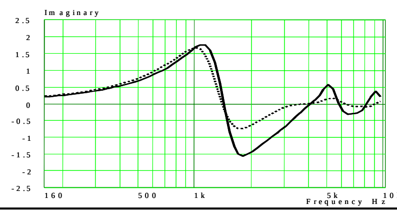
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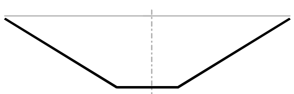

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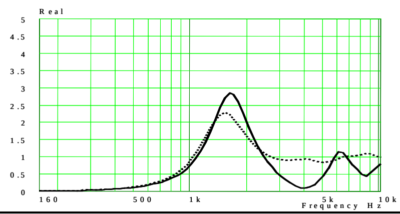
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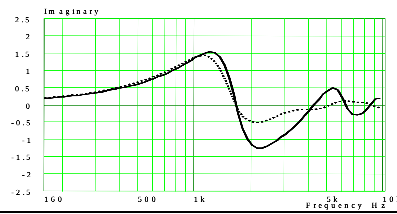
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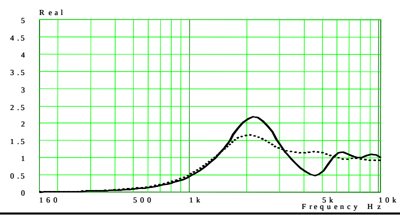
h = 25 mm

α = 74.05 º</td><td>

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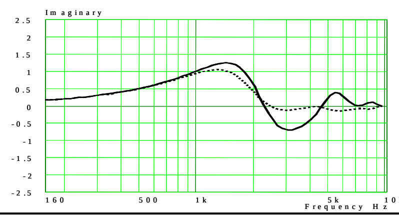
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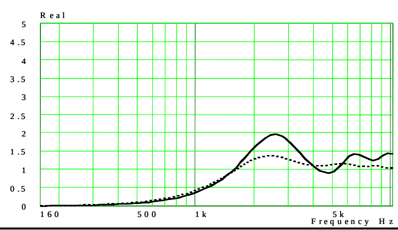
h = 12.5 mm

α = 81.87 º</td><td>

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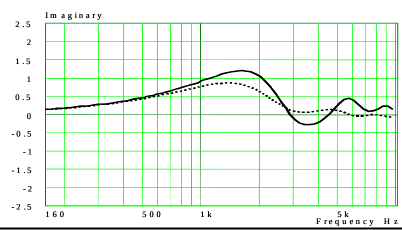
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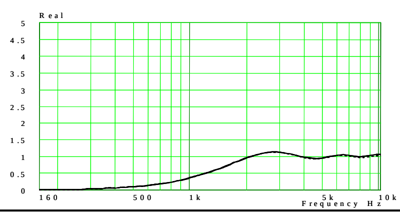
<tr><td>Disk for comparison

h = 0

α = 90 º</td><td>

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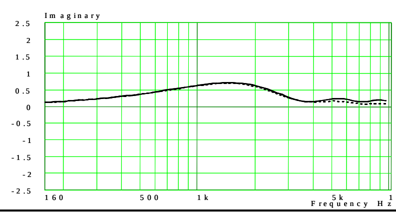
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